رياضيات هندسية (٥)
Second

73/100

تم الرفع بواسطة
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Engineering Math II

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Student Name(بالعربية)....[redacted] Time :1 hour

Question One: (25p)

Determine which of following statements true or false.

T 1. (~~T~~ F)The D.E with order n and constant coefficient has no singular point

$\frac{-2x^2}{x+3} =$

2. (F)The D.E (x+3) Y'' -2x Y' +(1-x^2)y=0 has irregular singular point at x=3

$X = -3$ Singular point $x = -3$

T 3. (~~T~~ F)If F(r)=r^2-2r+1 for an Euler equation then the W(y_1,y_2)=x x^{-2} x^{-1} $\frac{X\cancel{x}2X}{x+3}$ $\frac{2x^2}{x+3}$ $\frac{2x^2}{0}$

$(r-1)(r-1)$ ①

F 4. (~~F~~ T)The radius of convergence of series solution D.E
(4+x^2) Y''+2xy'+4x^2y about the point x=1 is 2.

5. (T)$\lim_{x\to0} xp = 3$ and $\lim_{x\to0} x^2q = -6$ then the D.E in Euler form is
$\frac{1}{3}x^2$ Y''+xy'-2y=0 $xp = \frac{x \cdot 3x}{x^2} = 3$ $\frac{x^2 \cdot -6}{x^2}$ $2r-1=0$ $r=0, \frac{1}{2}$

F 6. (~~F~~ T) If the indicial equation F(r)=2r^2-r then one of the solution is
For D.E is $1 + \sum_{n=1}^{\infty} a_n\,(0.5)X^n$

7. (F)Let f(t)=2+e^{-2t} then the Laplace transform $L(f(t))$=$\frac{4s+2}{s^2+2s}$ 17.5

8. (T)If F(s)= $\frac{4s}{s^2+16s}$ $\frac{4}{s+16}$ then $L^{-1}(F(s))=4e^{-16t}$;s>16

9. (F)f(t)=t , g(t)= 2t then the Laplace transform $L(f(t).g(t))$=$\frac{2}{s^3}$

10. (F) If y=$\varphi(x)$ *is a solution for second order D.E* near an ordinary point x=0 where y(0)=1, y/(0)=-1 y//(0)=-2then a_2=-0.5 $\phi'' = 2a_2$

11. (T) $\frac{cosx}{x-\frac{\pi}{2}}$is analytic around $x = \frac{\pi}{2}$

$\phi'' + \phi' + \phi = 0$ 9
$-2 \quad 1 \quad 1 \quad -2=0$

Question Two: (25p)

a) Determine a lower bounded for radius of convergence of series solution about the given point. $X=2$ (15p)

$(1+x^3)y''+4xy'+y=0$

$1+x^3=0 \Rightarrow x=\sqrt[3]{-1}=\pm i$

$1\pm i$

$z=\sqrt{0+1}=1 \Rightarrow \theta=\frac{\pi}{2}$

$\Rightarrow r^{1/3} e^{i\frac{\pi}{2}\left(\frac{\pi}{4}+\frac{2k\pi}{4}\right)}$ 0, 1

$= e^{\frac{i\pi}{2}}\left(\frac{\pi}{2}\right) \Rightarrow (\cos\frac{\pi}{2}+\sin\frac{\pi}{2}i)=1$

$e^{\frac{i\pi}{2}}\left(\frac{\pi}{4}+\frac{2\pi}{4}\right)=e^{\frac{i\pi}{2}}\left(\frac{3\pi}{4}\right)=\cos\frac{3\pi}{4}+\sin\frac{3\pi}{4}i$

$=-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$

for ①

$x=2$ $\sqrt{2^2+1^2}=\sqrt{5}$

$\left(\sqrt{(2-\frac{1}{\sqrt{2}})^2+\frac{1}{\sqrt{2}}}\right)=1.6+\frac{1}{2}=2.1 \Rightarrow$ the lower is $\boxed{2.1}$

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b) find all singular point of the D.E and Determine whether each one regular or irregular. (10p)

$(x+2)^2(x-1)y''+3(x-1)y'-2(x+2)y=0$

$=(x^2+4x+4)(x-1)y''+3(x-1)y'-2(x+2)y=0$

$(x+2)(x+2)$

$(X=-2,\ X=1)$ two singular point

$X=-2$

$\lim_{x\to-2}\frac{3x(\cancel{x-1})}{(x+2)^2\cancel{(x-1)}}=\lim_{x\to-2}\frac{3x}{(x+2)^2}=\frac{-6}{0}=\infty$ irrigular ~~singular point~~

$\lim_{x\to-2}\frac{x^2 * -2\cancel{(x+2)}}{(x+2)^{\cancel{2}}(x-1)}=\lim_{x\to-2}\frac{-2x^2}{(x+2)(x-1)}=\frac{-8}{0}=\infty$

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for $x=1$

$\lim_{x\to1}\frac{3x\cancel{(x-1)}}{(x+2)^2\cancel{(x-1)}}=\frac{3*1}{9}=\frac{3}{9}=\frac{1}{3}$

$\lim_{x\to1}\frac{x^2 * -2\cancel{(x+2)}}{\cancel{(x+2)}^2(x-1)}=\frac{-2(1)^2}{3(0)}=\frac{-2}{0}=\infty$

irrigular Singular Point

Question Three: (25p)

Find the solution for the given initial value problem

$2x^2 Y'' + x Y' - 3y = 0 \qquad y(1)=1, y'(1)=4$

$\Rightarrow = x^2 y'' + \frac{x}{2} y' - \frac{3y}{2} = 0$

$x = 0$ singular point

$P(x) = \lim_{x \to 0} \frac{x * x}{2x^2} = \frac{1}{2}$ finite

$q(x) = \lim_{x \to 0} \frac{x^2 * -3}{2x^2} = \frac{-3}{2}$ finite

$x = 0$ regular singular point

$\alpha = \frac{1}{2} \quad \beta = \frac{-3}{2}$

$r^2 + r(\alpha - 1) + \beta$

$= r^2 + r(0.5 - 1) + \frac{-3}{2}$

$= r^2 + r(\frac{-1}{2}) + \frac{-3}{2}$

$= 2r^2 - r - 3$

$(2r-3)(r+1)$

التأكد $2r^2 + 2r - 3r - 3 = 2r^2 - r - 3$

$r = -1, \frac{3}{2}$

$y = c_1 x^{-1} + c_2 x^{\frac{3}{2}}$

$y(1) = 1 \Rightarrow c_1 * 1 + c_2 * 1 = 1$

$\Rightarrow c_1 + c_2 = 1$ — 1

$y' = -c_1 x^{-2} + \frac{3}{2} c_2 x^{0.5}$

$y'(1) = 4 \Rightarrow -c_1 + \frac{3}{2} c_2 = 4$ — 2

$c_1 = 1 - c_2$

$\Rightarrow -(1 - c_2) + \frac{3}{2} c_2 = 4$

$-1 + c_2 + \frac{3}{2} c_2 = 4$

$c_2 + \frac{3}{2} c_2 = 5 \Rightarrow \frac{5}{2} c_2 = 5 \Rightarrow \boxed{c_2 = 2}$

$c_1 = 1 - c_2 = 1 - 2 = -1$

$\therefore c_1 = \boxed{-1}$

$c_2 = \boxed{2}$

الحل النهائي

$y = -x^{-1} + 2x^{\frac{3}{2}}$

$\boxed{y = \frac{-1}{x} + 2x^{\frac{3}{2}}}$

(scratch work:)

$(2r-3)(r-1)$

$2r^2 - 2r - 3r + 3$

$(2r-3)(r+1)$

$2r^2 + 2r - 3r - 3$

$2r^2 - r - 3$

$r = \frac{3}{2}$

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Question four: (25p)

Use Laplace transform to solve the given initial value problem.

$y''-2y'-3y=0$; $y(0)=2$, $y'(0)=0$

$(s^2Y(s)-sY(0)-Y'(0))-2(sY(s)-Y(0))-3Y(s)$

$s^2Y(s)-sy(0)$

$(s^2Y(s)-sY(0)-Y'(0))-2(sY(s)-Y(0))-3Y(s)=0$

$s^2Y(s)-sY(0)-Y'(0)-2sY(s)-2Y(0)-3Y(s)=0$

$\cancel{Y(s)(s^2}$

$Y(s)(s^2-2s-3)-2s+4=0$

$\Rightarrow Y(s)(s^2-2s-3)=2s+4$

$$Y(s)=\frac{2s+4}{s^2-2s-3}$$

$$\frac{2s+4}{s^2-2s-3}=\frac{A}{s-3}+\frac{B}{s+1}$$

$2s+4=(s+1)A+B(s-3)$

for $s=3$

$10=4A+0 \Rightarrow A=\frac{10}{4}=2.5$

for $s=-1$

$2=(0)A+-4B \Rightarrow B=\frac{-2}{4}=-\frac{1}{2}$

$$2s+4=\frac{2.5}{s-3}+\frac{-0.5}{s+1} \Rightarrow = 2.5e^{3t}+0.5e^{-t}$$

2 2

$y'' - 2y' - 3y = 0$

$s^2 Y(s) - sy(0) - y'(0) - 2(sY(s) - \cancel{y(0)}) - 3Y(s)$

$s^2 Y(s) - sy(0) - \cancel{y'(0)} - 2sY(s) + 2y - 3Y(s)$

$Y(s)(s^2 - 2s\cancel{y(s)} - 3) = 2s\cancel{y(0)} \cancel{-4}$

$Y(s) = \dfrac{2s-4}{s^2-2s-3} \rightarrow \dfrac{2s-4}{(s+1)(s-3)}$

$2s - 4 = \dfrac{A}{s-3} + \dfrac{B}{s+1}$

$2s - 4 = A(s+1) + B(s-3)$

$s = 3 \Rightarrow$

$2 = 4A + 0$

$A = \frac{1}{2}$

$\cancel{s=1}$

$\cancel{-6} = -4B \Rightarrow B = 1.5$